# **Central University of Rajasthan** School of Mathematics, Statistics & Computational Sciences Department of Mathematics

# Details of Revised/New Structure and Syllabus of Ph.D. Mathematics w.e.f. 2023

# **POs (Programme Outcomes)**

## Students should be able to:

**PO 1.** Rigorous knowledge: Having rigorous and thorough knowledge of the broad research area.

**PO 2.** Application of knowledge: Apply the knowledge of area of research to the solution of complex mathematical problems.

**PO 3.** Problem analysis: Identify, formulate and analyze complex mathematical problems.

**PO 4.** Development of solution: Formulate solutions for the complex mathematical problems, process, and its components.

**PO 5:** Conduct investigation of complex problems: Use research methods including design of experiment, analysis and observation of results to investigate and solve complex problems.

**PO 6:** Tool and software usages: Create, select and apply appropriate mathematical techniques, resources and software tools.

**PO 7:** Environment and sustainability: Understand the role of mathematics and its impact in societal and environmental contexts and demonstrate the knowledge of, and need for sustainable development.

**PO 8:** Individual and teamwork: Function effectively as an individual and as a member of team.

**PO 9:** Life-long learning: Recognize the need for and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change.

**PO 10.** Carrier Opportunities: Having opportunity to start career in academics and research institutions

# **Mapping from Mission to Programme Outcomes**

The mapping is based on marks 1 to 3, where "1" indicates low level matching of mission with programme outcome, "2" indicates medium level matching, and "3" indicate high level matching.

PO/Mission	M 1	M 2	M 3
PO 1	3	3	3
PO 2	3	3	3
PO 3	2	3	2
PO 4	2	3	3
PO 5	2	3	3
PO 6	3	3	3
PO 7	3	2	2
PO 8	3	2	2
PO 9	3	2	2
PO 10	2	3	3

## Mapping from Programme Outcomes to Courses

The mapping is based on marks 1 to 3, where "1" indicates low level matching of course outcome with programme outcome, "2" indicates medium level matching, and "3" indicate high level matching.

	PO	РО								
Course/PO	1	2	3	4	5	6	7	8	9	10
<b>MAT700</b>	3	2	1	1	1	3	1	1	3	3
<b>MAT701</b>	3	3	3	2	2	1	1	2	2	3
MAT702	3	3	3	2	2	1	1	2	2	3
MAT703	3	3	3	2	3	1	1	2	2	3
<b>MAT704</b>	3	2	2	2	2	3	1	2	3	3
MAT705	3	2	2	2	2	3	1	2	3	3
<b>MAT706</b>	3	3	3	2	2	2	2	2	2	3
<b>MAT707</b>	3	3	2	2	2	1	1	2	3	3
<b>MAT708</b>	3	3	3	3	2	1	1	2	2	3
<b>MAT709</b>	3	2	2	3	2	1	1	2	2	3
<b>MAT710</b>	3	3	2	2	2	1	1	2	2	3
MAT711	3	3	2	2	2	1	1	2	2	3
MAT712	3	3	3	3	2	1	1	2	2	3
MAT713	3	3	3	3	2	2	1	2	2	3
MAT714	3	3	3	3	2	1	2	2	3	3

<b>MAT715</b>	3	3	3	3	3	1	1	2	2	3
<b>MAT716</b>	3	3	3	2	2	1	1	2	2	3
<b>MAT717</b>	3	3	3	3	2	1	1	2	2	3
<b>MAT718</b>	3	3	3	3	2	1	2	2	2	3
<b>MAT719</b>	3	3	3	2	1	1	1	2	3	3
<b>MAT720</b>	3	3	3	3	2	1	1	2	3	3
MAT721	3	3	3	3	2	1	2	2	2	3
MAT722	3	3	3	3	2	1	1	2	2	3

# Level-7 (Ph.D.)

S. No.	Course Code (Tentative)	Course Name	Credit	Course Proposed/Revised by	Done/NOT Done
1	MAT-700	RESEARCH METHODOLOGY IN MATHEMATICS	04	Dr. Jai Prakash Tripathi	Done
2	MAT-701	ALGEBRA-I	04	Dr. Vipul Kakkar	Revised/Core Course
3	MAT-702	ANALYTIC FUNCTIONS THEORY & SPECIAL FUNCTIONS	04	Prof. J. K. Prajapat	Revised/Core Course
4	MAT-703	CATEGORY THEORY	04	Dr. Vijay Kumar Yadav	New/Core Course
5	MAT-704	COMPUTATIONAL METHODS: ALGORITHMIC APPROACH	04	Dr. Asha Kumari Meena	New/Core Course
6	MAT-705	COMPUTATIONAL FLUID DYNAMICS	04	Dr. Anand Kumar	Revised/Core Course
7	MAT-706	DYNAMICAL SYSTEMS	04	Dr. Ram Kishor/Dr. Jai Prakash Tripathi	Done
8	MAT-707	OPTIMIZATION TECHNIQUES	04	Prof. D.C. Sharma/Dr. Vidyottama Jain	Revised/Core Course

9	MAT-708	SOLID MECHANICS	04	Dr. Kamlesh Jangid	New/Core Course
10	MAT-709	ALGEBRA-II	04	Dr. Vipul Kakkar	Revised/Core Course

11	MAT-710	CELESTIAL MECHANICS AND ITS METHODS	04	Dr. Ram Kishor	Revised/Core Course
12	MAT-711	ANALYTIC FUNCTIONS THEORY & SPECIAL FUNCTIONS	04	Prof. J. K. Prajapat	Revised/Core Course
13	MAT-712	FUZZY SET THEORY, FUZZY LOGIC AND APPLICATIONS	04	Dr. Vidyottama Jain/Dr. Vijay Kumar Yadav	Revised/Core Course
14	MAT-713	MAGNETOHYDRODYNAMI CS	04	Dr. Anand Kumar	Revised/Core Course
15	MAT-714	MATHEMATICAL BIOLOGY	04	Dr. Jai Prakash Tripathi	Revised/Core Course
16	MAT-715	NUMERICAL METHODS FOR HYPERBOLIC PDES	04	Dr. Asha Kumari Meena	New/Core Course
17	MAT-716	QUEUEING THEORY	04	Prof. D. C. Sharma/Dr. Vidyottama Jain	Revised/Core Course
18	MAT-717	HOLOMORPHIC DYNAMICS	04		
19	MAT-718	APPLIED DYNAMICAL SYSTEMS	04	Dr. Jai Prakash Tripathi	Done
20	MAT-719	THERMAL INSTABILITIES AND METHODS	04	Dr. Anand Kumar	
21	MAT-720	COMPLEX DYNAMICS	04	Prof. J. K. Prajapat	
22	MAT-721	FRACTURE MECHANICS	04	Dr. Kamlesh Jangid	New/Core Course
23	MAT-722	Fuzzy Automata and Languages: Theory and Applications	04	Dr. Vijay Kumar Yadav	New/Core Course

	Course Title	<u>Course-Code: MAT700</u> RESEARCH METHODOLOGY	IN MATHEMATICS		
Teaching Scl	heme	Examination Scheme	Credits Allotted		
Theory: 3 hou	urs/ week	ESE: 60 Marks Internal Assessment: 40 Marks	Theory: 3		
Tutorial: 1 Ho	our/Week		Tutorial: 1		
			Total: 4		
Course Prer	equisite:				
	Basic knowledge of computer typing				
Course Obje	ctive:				
1.	To introduce computing softwares like Mathematica and MATLAB along with Latex a document preparation system				
2.	To introduce different significant information related to research, e.g., about writing research papers, different online searchable databases etc.				
3.	To demonstra objectives	te the ability to choose methods appr	ropriate to research aims and		
4.	to demonstra	te advanced writing skills			
Course Outc	omes: Students	will be able to			

1.	formulate the research problems.				
2.	use mathscinet, web of science and Google scholar etc.				
3.	create research articles in LaTex.				
4.	work on Matlab and Mathematica				
Course Conto	ent:				
Unit-I	Introduction of research, Importance of research, Research methods and research methodology, Importance of research methodology, Types of research, Various stages of research, Selection of research topic, Identification of research problems, Definition and formulation of the problem, Literature survey, Internet as a medium for research, Knowledge of web search:, Elements of an article: Title, Abstract, Keyword, Introduction, Formulation, Result and discussion, References	15 Hours			
Unit-II	Evaluation of research: plagiarism, citation, impact factor etc. Review of research papers, Working knowledge of Google Scholar, Research Gate, Web of Science, MathSciNet, SCOPUS and/or other open- source/subscribed journals and books, Introduction:- Latex and open office, Writing of simple articles, letters and applications, Mathematical symbols and commands, arrays, formulas and equations, Spacing, Borders and Colors,Creating different templates	15 Hours			
Unit-III	Writing of research articles, reports etc. Preparation of template of thesis and books. Preparation of ppt. poster, etc., Pictures and Graphics	15 Hours			
Unit-IV	MATLAB and Mathematica: Basic introduction:	15 Hours			

	Arithmetic operations, functions, plotting the graphs of different functions, Matrix operations, finding roots of an equation, finding roots of a system of equations, solving differential equations. Basic 2-D plots and 3- Dplots.			
Internal Asse	essment:			
CIA*-1	Written Exam.			
CIA-II	Written Exams/ Quizzes /Assignment /Presentations/ Viva-Voce			
ESE**	Written Exam. for Unit-I, II, III, & IV			
*: Continuous Internal Assessment **: End of Semester Examination				
Text Books:				

<sup>1.</sup> Pratap R., 2010, Getting Started with MATLAB, Oxford University Press.

2. Lynch S., 2014, Dynamical Systems with Applications using MATLAB, Birkhäuser.

3. Abell M.L. and Braselton J.P., 2004, *Differential Equations with Mathematica*, Elsevier Academic Press,.

4. Stavroulakis I. P. and Tersian S.A., 2004, An Introduction with Mathematica and MAPLE, World Scientific.

5. Lamport L.W., 1994, *LaTeX: A Document Preparation Systems, Addison-Wesley* Publishing Company.

### **Reference Books:**

6.Kopka H. and Daly P.W., 2004, Guide to LATEX (4th Ed.), Addison Wesley.

		<u>Course-Code: MAT701</u> <u>Course Title:</u> ALGEBRA-I			
Teaching Scl	heme	Examination Scheme	Credits Allotted		
Theory: 3 hou	urs/ week	ESE: 60 Marks Internal Assessment: 40 Marks	Theory: 3		
Tutorial: 1 Ho	our/Week		Tutorial: 1		
			Total: 4		
Course Prer	equisite:				
	Knowledge	of groups			
Course Obje	ctive: To intro	luce			
1.	Presentation	Presentation of groups			
2.	Coset enume	Coset enumeration method			
3.	Products of g	roups			
4.	Non-associative structure				
Course Outc	omes: Students	s will be able to understand			

1.	Dehn's Fundamental problem				
2.	Free product of groups				
3.	Semidirect product of groups				
4.	Quasigroups and Loops				
Course Conte	ent:				
Unit-I	Free groups, Presentation of groups, Generators and relations, Finitely generated groups, Dehn's Fundamental problem, Word's problem, Conjugacy problem, Isomorphism problem,.	15 Hours			
Unit-II	Canonical form of word, Coset enumeration method, Schreier's transversal, Free product of groups,	15 Hours			
Unit-III	Central product of groups, Amalgamation, Semidirect product of groups, Classification of groups of order pq, Affine groups,	15 Hours			
Unit-IV	Non-associative algebraic structures, Universal Algebra, Quasigroups, Loops, Right loops, Multiplication group of loops, Inner mapping groups, Transversal in groups as algebraic structure, Congruences in right loops, Maltcev variety	15 Hours			
Internal Asse	essment:				
CIA*-1	Written Exam.				
CIA-II	Written Exams/ Quizzes /Assignment /Presentations/ Viva-Voce				

ESE**	Written Exam. for Unit-I, II, III, & IV					
	bus Internal Assessment Semester Examination					
Text Book	5:					
<ol> <li>2. La</li> <li>3. Sn</li> <li>4. Ma</li> </ol>	<ul> <li>bbinson D., 1996, A Course in the Theory of Groups, Springer.</li> <li>1 R., 2017, Algebra I &amp; II, Springer.</li> <li>nith J. D. H., 2006, An Introduction to Quasigroups and Their Representations, CRC Press.</li> <li>Ignus W., Karrass A., and Solitar D., 1976, Combinatorial Group Theory, Dover plication.</li> </ul>					
Refere	nce Books:					
5. Suz	uki M., 1982, Group Theory I & II, Springer Verlag.					
E-resource	28:					

<u>Course-Code:</u> MAT702 <u>Course Title:</u> ANALYTIC FUNCTIONS THEORY & SPECIAL FUNCTIONS		
Teaching Scheme	Examination Scheme	Credits Allotted

Theory: 3 ho	ours/ week	ESE: 60 Marks Internal Assessment: 40 Marks	Theory: 3
Tutorial: 1 Hour/Week			Tutorial: 1
			Total: 4
Course Prei	requisite:		
	Basics of Rea	al and Complex Analysis.	
Course Obj	ective: To devel	op the concept of	
1.	special funct	aims to demonstrate the fundamentals of tions including Gamma functions, Hype Legendre polynomials.	
Course Out	comes: Students	will be able to	
1.		rse students are fitted with basic theory, anal nctions and special functions.	yze properties and applications
Course Con	tent:		
Unit-I	mappings, theorem, A formula, D	nctions, Schwarz lemma, Conformal Open mapping theorem, Hurwitz' nalytic continuation, Poisson integral irichlet problem, Zeros of analytic ouche's theorem.	
Unit-II	of entire symbols o function, A	luct of analytic functions, Factorization functions, Gamma functions, Order and O, Beta functions, Factorial symptotic expansion, Riemann Zeta teneralizations of Riemann zeta	

	functions.	
Unit-III	Hypergeometric Function, Elementary Properties, Contiguous function relations, Integral Representation, Simple transformation, Quadratic transformation, Generalized Hypergeometric Functions, Bessel functions, Elementary Properties.	15 Hours
Unit-IV	Order of entire functions, Univalent functions, Basic results of univalent functions, Normal families, Riemann mapping theorem, Area theorem, Biberbach theorem, Koebe 1/4 theorem.	15 Hours
Internal A	ssessment:	
CIA*-1	Written Exam.	
CIA-II	Written Exams/ Quizzes /Assignment /Presentations/ Viva-Voce	
ESE**	Written Exam. for Unit-I, II, III, & IV	
	bus Internal Assessment Semester Examination	
Text Book	s:	
2. Ra 3. Be	nnusamy S., 2005, Foundations of Complex Analysis, Na inville E. D., 1960, Special Functions, The MacMillan C Il W.W., 1968, Special Functions for Scientists and Engi mp. Ltd.	omp.

	Course Code: MAT703		
		<u>Course Title:</u> CATEGORY THEORY	Y
Teaching	g Scheme	Examination Scheme	Credits Allotted
Theory: 3	3 hours/ week	ESE: 60 Marks Internal Assessment: 40 Marks	Theory: 3
Tutorial:	1 Hour/Week		Tutorial: 1
			Total: 4
Course I	Prerequisite: None	e	I
Course (	Objective:		
1	-	n array of tools for stating results which ca well as computer science spectrum.	n be used across a wide
2	To express key o	concepts in terms of mappings rather than in	n terms of set elements.
3	To develop und languages.	erstanding how categorical concepts can be	e used in automata theory and
Course (	<b>Outcomes:</b> The stu	dents will be able to understand	
1.		ons of mathematical structures find conven will be able to use functors as the appropri	

	different domains of mathematical discourse.	
2	the use of functors as the appropriate tools to compare different domain discourse and will be able to absorb the concepts and topics in hand with	
3	students will be able to understand the significance of the concepts defined and the theorems proved here;	
4	The course will serve as a foundation for application of categorical con- domains of mathematics and computer science in a generalized form to and vagueness involved in complex systems.	•
Course C	ontent:	
Unit-I	Monomorphisms, Epimorphisms, and Isomorphisms, Basic concepts of category theory: Categories, Duality principle for category theory, Subcategories, Diagrams, Monomorphisms, Epimorphisms, and Isomorphisms in a category, Initial and terminal objects, Zero object, with examples.	15 Hours
Unit-II	Products, Coproducts, Relations as a pair of maps, Equalizers and coequalizers, Pullbacks, Limits and colimits. Functors; identity, constant, forgetful, faithful, full and amnestic functor; covariant and contravariant hom-functor, covariant and contravariant power-set functor.	15 Hours
Unit-III	Free and cofree object, Left adjoint and right adjoint of a functor, Yoneda Proposition, Perseverance of products and equalizers by any functor with a left adjoint and its converse, Adjoint functor, Adjoint functor theorem.	15 Hours
Unit-IV	Constructs, Concrete categories, Concrete functors, Concrete isomorphisms, Duality for concrete categories, Natural	15 Hours

	transformations, Natural isomorphisms and adjunctions, Yoneda Lemma, Compositions of Natural Transformations, Sources and Sinks, Factorization structures.	
Internal	Assessment:	
CIA*-1	Unit -I	
CIA-II	Written Exams/ Quizzes /Assignment /Presentations/ Viva-Voce	
ESE**	Unit-I,II,III,IV	
	uous Internal Assessment f Semester Examination	
Text Boo	ks:	
0. 2. Ar	<ul> <li>damek J., Herrlich H. and Strecker G. E., 2006, Abstract and Concrete C. f Cats, Dover Publications.</li> <li>bib M. A. and Manes E. G., 1975, Arrows, Structures &amp; Functors: The Comperative, Academic Press.</li> </ul>	
	enjamin Pierce, Basic Category Theory for Computer Scientists. Cambridge	. MIT Press
4. St	even Awodey, Category Theory. Oxford University Press, 2006.	
Referenc	e Books:	
5. La	ane S. M., 1971, <i>Categories for the Working Mathematician (2<sup>rd</sup> Ed.)</i> , Spr	inger, New York.
E-resour	ces:	

Teach	ing Scheme	Examination Scheme	Credits Allotted
Theor	y: 3 hours/ week	ESE: 60 Marks Internal Assessment: 40 Marks	Theory: 3
Tutori	al: 1 Hour/Week		Tutorial: 1
			Total: 4
	se Prerequisite: Nur	merical Methods.	
Cours	se Objective:		
1	This course is hyperbolic PD	designed to introduce the algorithmic approaces.	h of the numerical methods fo
2	First two units	are designed to develop the understanding of a	algorithmic approaches.

1.	to code a numerical method from numerical analysis;		
2	the use of numerical method from numerical analysis in solving a hyperbolic PDEs;		
3	to code the computational methods to solve hyperbolic PDEs.;		
4	students will be able to code the numerical methods and compute the for the given problems.	numerical solutions	
Course C	ontent:		
Unit-I	Review of numerical analysis, Algorithmic approach for the root finding numerical methods, algorithmic approach for the numerical methods to solve the system of linear equations.	15 Hours	
Unit-II	Algorithmic approach for numerical differentiation and numerical integration methods, Algorithm approach for the numerical methods to solve the ordinary differential equations.	15 Hours	
Unit-III	Algorithmic approach of the first order finite volume schemes for conservation laws: Lax-Friedrich scheme, Roe Scheme, Godunov scheme, Rusanov scheme.	15 Hours	
Unit-IV	Algorithmic approach of second order finite volume schemes for conservation laws: Lax-Wendroff scheme, MUSCL scheme - minmod limiter, superbee limiter.	15 Hours	
Internal A	Internal Assessment:		
CIA*-1	Unit -I		
CIA-II	Written Exams/ Quizzes /Assignment /Presentations/ Viva-Voce		

ESE**	Unit-I,II,III,IV	
	nous Internal Assessment f Semester Examination	
Text Boo	ks:	
	nte S. D. and Cart D. B., 1980, <i>Elementary Numerical Analysis: An Alg</i> <sup>rd</sup> Ed.), McGraw-Hill.	orithmic Approach
	kkipati R. V., 2010, <i>MATLAB An introduction with Applications</i> , New A imited, Publishers.	ge International (P)
Referenc	e Books:	
	sthaven J. S., 2018, Numerical Methods for Conservation Laws: From Igorithms, SIAM.	e Analysis to
E-resour	ces:	

# Course-Code: MAT-705

### Course Title: COMPUTATIONAL FLUID DYNAMICS

Teaching	Scheme	Examination Scheme	Credits Allotted
Theory: 3	3 Hours/week	ESE: 60 Marks Internal Assessment: 40 Marks	Theory: 3
Tutorial:	1 Hour/week		Tutorial: 1
			Total: 4
Course P	rerequisite: The student	should have knowledge of	
1.	Basic Concepts of Nur	merical Methods.	
2.	Basic Concepts of Flu	id Dynamics.	
Course O	bjective: This course aim	s to learn	
1.	the basics of different	numerical techniques.	
2.	the convergence and s	tability of different methods.	
3.	the implementation of	numerical techniques to fluid dynamics	
4.	the numerical solution	by using computer programming.	
Course O	Uutcomes: After studying	this course the student will be able to	
1.	the concepts of consist	tency, stability and convergence analysis	s.

2.	check the stability and convergence of numerical methods.	
3.	solve numerically differential equations of fluid dynamics.	
4.	find the numerical solution to fluid dynamics by using a computer program.	
Course Co	ntent:	
Unit-I	Simpson's method, integration with equal and unequal segments, Euler's method, Runge-Kutta method, shooting method, bisection method, grid generation, Self-similarity solutions of laminar boundary layer flows	15 Hours
Unit-II	radial flow caused by distributed sources and sinks. numerical methods for solving elliptic and hyperbolic partial differential equations,	15 Hours
Unit- III	explicit and implicit methods for solving parabolic partial differential equations, starting flow in channel, pipe and open channel flow, potential flows in ducts, generalized Rayleigh problem, solution of Bi-harmonic equations, Stokes flows.	15 Hours
Unit- IV	Numerical computation of the problems based on the above methods using a programming language or software.	15 Hours
Internal A	ssessment:	
CIA-I*	Unit-I	
CIA-II	Written Exams/ Quizzes/ Assignment/ Presentations/ Viva-Voce	

ESE**	Unit-I, II, III, IV	
*: Continuous Internal Assessment **: End of Semester Examination		
Text Book	۲S:	
1. Cha	w C., 1979, An Introduction to Computational Fluid Mechanics, John Wiley & So	ons.
2. Flete	cher C. A. J., 2003, Computational Techniques for Fluid dynamics, Vol. 1, Spring	ger.
3. Fleto	cher C. A. J., 2003, Computational Techniques for Fluid dynamics, Vol. 2, Spring	ger.
4. Bose	e T. K., 1997, Numerical Fluid Dynamics, Narosa Publishing House.	

5. Sherman F. S., 1990, Viscous Flow, McGraw-Hill Publishing.

### **Reference Books:**

1. Anderson J. D., 1995, Computational Fluid Dynamic: The Basics with Applications, McGraw-Hill, Inc.

2. Chung T.J., 2014, *Computational Fluid Dynamics*, Ed. 2<sup>nd</sup>, Cambridge university Press.

### **E-resources:**

https://onlinecourses.nptel.ac.in/noc20\_ch05/preview

<u>Course-Code: MAT 706</u> <u>Course Title: Dynamical Systems</u>			
Teaching Scheme	Examination Scheme	Credits Allotted	
Theory: 3 hours/ week	ESE: 60 Marks Internal Assessment: 40 Marks	Theory: 3	
Tutorial: 1 Hour/Week		Tutorial: 1	
		Total: 4	
Course Prerequisite: The student should have knowledge of			

1.	Differential Equations, Analysis and Linear Algebra and Real Analysis, Geometry
Course Obje	ctive:
1.	To introduce the theory, properties and applications of various dynamical systems
2.	To make the students familiar with theory of linear and nonlinear dynamical systems
3.	To introduce Hartman-Grobman theorem and stable manifold theorem
4.	To introduce center manifold theory and normal form theory
5.	To introduce different aspects associated with discrete dynamical systems
Course Outc	omes: The students will be able to
1.	understand important aspects such as well posedness (existence, uniqueness and stability) associated with a linear systems
2.	understand important aspects such as well posedness (existence, uniqueness and stability) associated with a non-linear systems
3.	learn different dynamical systems tools e.g., Hartman-Grobman, stable manifold theorems, center and normal form theory
4.	To analyze discrete dynamical systems and their stability
5.	how to apply these concepts in different fields like nonlinear dynamics, population

	dynamics, chemical kinetics, epidemiology, environmental sciences etc.		
Course Co	ntent:		
Unit-I	Nonlinear Systems- local analysis: the fundamental existence-uniqueness theorem, The flow defined by a differential equation, Linearization, The stable manifold theorem, The Hartman-Grobman theorem,	15 Hours	
Unit-II	Stability and Lyapunov functions, Saddles, Nodes, Foci, and Centers, Nonlinear Systems- global analysis: Dynamical systems and global existence theorem, Limit sets and Attractors, Periodic orbits, Limit Cycles, and Separatrix cycles, the Poincare map.	15 Hours	
Unit-III	the stable manifold theorem for periodic orbits, the Poincare-Bendixson theorem in in xy-plane, Linear Systems, Bendixon's Criterion, Discrete dynamical systems: finite dimensional maps, limit sets, Stability, Invariant manifolds	15 Hours	
Unit-IV	Runge-Kutta methods: the framework, linear decay, Lipschitz conditions, Dissipative systems, generalized dissipative systems, Gradient system	15 Hours	
Internal As	ssessment:		
CIA*-1	Unit -I		
CIA-II	Written Exams/ Quizzes /Assignment /Presentations/ Viva-Voce		
ESE**	Unit-I, II, III, IV		
	bus Internal Assessment Semester Examination	I	

**Text Books:** 

1. Perko L., 2006, Differential Equations and Dynamical Systems, Springer-Verlag.

- 2. Stuart A. M. and Humphries A. R., 1998, *Dynamical Systems and Numerical Analysis*, Cambridge University Press.
- 3. Hirsch M. W., Smale S. and Robert L.D., 2013, *Differential Equations, Dynamical Systems and An Introduction to Chaos*, Academic Press.
- 3. Lynch S., 2004, Dynamical Systems with Applications using MATLAB, Birkhause Press.

#### **Reference Books:**

4. Strogatz, S. H., 2000, Nonlinear Dynamics and Chaos with Applications to Physics, Biology, Chemistry and Engineering, Westview Press.

#### **E-resources:**

https://www.youtube.com/playlist?list=PLbN57C5Zdl6j\_qJA-pARJnKsmROzPnO9V

https://www.youtube.com/watch?v=BRaliLNuvNg&list=PL6hB9Fh0Z1ELbHIAL22dCk173qykDgeoz

<u>Course-Code: MAT707</u> <u>Course Title: OPTIMIZATION TECHNIQUES</u>		
Teaching Scheme	Examination Scheme	Credits Allotted

Theory: 3 hou	rs/ week	ESE: 60 Marks Internal Assessment: 40 Marks	Theory: 3
Tutorial: 1 Ho	our/Week		Tutorial: 1
			Total: 4
Course Prere	equisite:		
	Basics of Ma	thematical Programming.	
Course Obje	ctive:		
1.	This course focuses on different optimization techniques to solve one dimensional unconstrained optimization problems.		
2.	This course focuses on different optimization techniques to solve unconstrained optimization problems.		
3.	This course focuses on various optimization techniques to solve constrained linear and nonlinear optimization problems.		
4.	This course focuses on evolutionary optimization methods.		
Course Outco	Course Outcomes:		
1.	The students will be equipped with the knowledge of one dimensional optimization techniques such as region elimination techniques and so on.		
2.	The students will have a deep knowledge of various unconstrained optimization techniques.		

3.	The students will be able to solve linear and nonlinear constrained optimization techniques.			
4.				
	They will be having knowledge of evolutionary optimization	ion techniques.		
Course Co	ntent:			
Unit-I	One Dimensional Unconstrained Minimization: Single variable minimization, Unimodality, Region elimination techniques, Fibonacci Method, Golden section search method, Polynomial based methods, Optimization of non-unimodal functions.	15 Hours		
Unit-II	Unconstrained Optimization Methods: Steepest descent method, conjugate gradient method, Newton's method, Quasi-Newton's method, steepest descent method, direct search method.	15 Hours		
Unit-III	Constrained optimization method: Linear and Nonlinear Programming Problems, Simplex Method, Fritz John and Karush-Kuhn-Tucker optimality conditions, Convex Optimization, The concept of Penalty functions and Barrier functions, algorithm and their global convergence.	15 Hours		
Unit-IV	Evolutionary based local and global optimization methods: genetic algorithm, ant colony optimization, particle swarm method, simulated annealing, harmony search algorithm, stochastic gradient method and their applications to real life problems.	15 Hours		
Internal As	ssessment:			
CIA*-1	Written Exam.			
l				

CIA-II	Written Exams/ Quizzes /Assignment /Presentations/ Viva-Voce	
ESE**	Written Exam. for Unit-I, II, III, & IV	
	Internal Assessment mester Examination	

### **Text Books:**

- 1. Bertsekas D., 1999, Nonlinear Programming (2<sup>nd</sup> Ed.), Athena Scientific, Nashua.
- 2. Chandra S., Jayadeva and Mehra A., 2009, *Numerical Optimization with Applications*, Narosa Publishing House Pvt. Ltd.
- 3. Fletcher R., 2000, Practical Methods of Optimization, John wiley.
- 4. Luenberger D. and Ye Y., 2008, *Linear and Nonlinear Programming* (3<sup>rd</sup> Ed.), Springer.
- 5. Mangasarian O. L., 1987, Nonlinear Programming, SIAM, Philadelphia.
- 6. Nocedal J.and Wright S., 1999, Numerical Optimization, Springer-Verlag, New York.
- 7. Simon D., 2013, Evolutionary Optimization Algorithms, Wiley.

#### **Reference Books**

- 1. Nocedal J.and Wright S., 1999, Numerical Optimization, Springer-Verlag, New York.
- 2. Mangasarian O. L., 1987, Nonlinear Programming, SIAM, Philadelphia.

- 1. https://archive.nptel.ac.in/courses/112/103/112103301/
- 2. https://archive.nptel.ac.in/courses/111/105/111105100/

<u>Course-Code: MAT708</u> Course Title: SOLID MECHANICS		
Teaching Scheme	Examination Scheme	Credits Allotted
Theory: 3 hours/ week	ESE: 60 Marks Internal Assessment: 40 Marks	Theory: 3

Tutorial: 1	Hour/Week		Tutorial: 1
			Total: 4
Course Pro	erequisite: The st	udent should have knowledge of	1
1.	Basics of Me	chanics.	
Course Ob	ojective:		
1.	To introduce t	he concepts of equilibrium in components	
2.	To introduce t	he concepts of deformation in components	
3.	To introduce t	he concepts of structures for engineering designs	
Course Ou	<b>itcomes:</b> at the end	d of this course, students will be able to	
1.	explain stress,	strain and deformations in the materials	
2.	apply Hooke's	apply Hooke's law for isotropic materials	
3.	classify the hardness and strength of the materials		
4.	explain brittle and ductile materials.		
Course Co	ntent:		
Unit-I	components of equality of sh	ress, normal stress and shear stress, nine Cartesian of stress at a point, sign convention and notation, ear stresses on mutually perpendicular planes and faction, principal stresses, stress circle.	
Unit-II	of strain, Poi strain circle; materials, ela	rain, normal and shear strain, two-dimensional state isson's ratio, volumetric strain, principal strains, Hooke's law and its application to isotropic stic constants and their relationships, plane stress in conditions, strain energy density.	

Unit-III	-III Uniaxial tension test to determine yield and ultimate strength of 15 Hours materials, stress-strain diagram, proof stress, ductile and brittle materials, hardness and impact strength.	
Unit-IV	Unit-IV Conditions affecting mechanical behaviour of engineering materials; Stresses and strains in thin cylindrical shells and spheres under internal pressure.	
Internal As	sessment:	
CIA*-1	Written Exam.	
CIA-II	Written Exams/ Quizzes /Assignment /Presentations/ Viva- Voce	
ESE**	Written Exam. for Unit-I, II, III, & IV	
	us Internal Assessment emester Examination	
Text Books	:	
2. Tim	ov, E. P., 2015, <i>Engineering Mechanics of Solids</i> , Pearson (Second ed oshenko, S.P. and Young, D.H., 2009, <i>Elements of Strength of Mate</i> MKS Units), East-West Press Pvt. Ltd.	
Reference I	Books	
	J.M. and Goodno, B.J., 2009, <i>Strength of Materials</i> , Indian Edition (4 rning India Private Ltd.	4th reprint), Cengage
E-resources	:	

Course-Code: MAT709	
Course Title: ALGEBRA-II	

Teaching Sch	Scheme Examination Scheme Cu		Credits Allotted
Theory: 3 hou	irs/ week	ESE: 60 Marks Internal Assessment: 40 Marks	Theory: 3
Tutorial: 1 Ho	our/Week		Tutorial: 1
			Total: 4
Course Prere	equisite:		
	Knowledge o	f groups and rings	
Course Obje	ctive: To introd	luce the students	
1.	The bilinear maps		
2.	The concepts of group extension		
3.	Equivalence of group extensions		
4.	Commutator subgroups		
Course Outc	omes: The stud	ents will be able to understand	
1.	Tensor Product of Algebras		
2.	The interpretation of first cohomology group		
3.	How to calculate the second cosmology groups of some finite groups of small order		

4.	Structure of finite p-groups upto order p^4.			
Course Content:				
Unit-I	Bilinear Maps, Tensor Product of Algebras, Exact sequences, Chain complex, Cochain complex, Boundary and coboundary, Cycle and cocycle.	15 Hours		
Unit-II	Ext and Tor, Homology and cohomology of chain complex, Long exact sequence, Connecting Homomorphism, Group Extension, Crossed homomorphism, principle crossed homomorphism, Interpretation of first cohomology group,	15 Hours		
Unit-III	Factor system, Equivalence of group extensions, Interpretation of second cohomology group, Schur- Zassenhaus Theorem, Calculation of second cosmology groups of some finite groups of small order	15 Hours		
Unit-IV	Commutator subgroups, Commutator identities, Higher commutator, Three subgroup lemma, Nilpotent Groups, Lower Central Series, Upper Central Series, Nilpotency class, finite p-groups, Structure of finite p-groups upto order p^4.	15 Hours		
Internal As	ssessment:			
CIA*-1	Written Exam.			
CIA-II	Written Exams/ Quizzes /Assignment /Presentations/ Viva-Voce			
ESE**	Written Exam. for Unit-I, II, III, & IV			
	ous Internal Assessment Semester Examination	1		

### **Text Books:**

1. Robinson D., 1996, A Course in the Theory of Groups, Springer.

2.Lal R., 2017, Algebra I & II, Springer.

3.Babakhanian A., 1972, Cohomological Methods in Group Theory, Marcel Dekker.

4. Vermani L. R., 2003, An Elementary Approach to Homological Algebra, CRC Press.

### **Reference Books**

1. Suzuki M., 1982, Group Theory I & II, Springer Verlag.

<u>Course-Code: MAT710</u> Course Title: Celestial Mechanics and Its Methods					
Teaching Scheme		Examination Scheme	Credits Allotted		
Theory: 3 hours/ week		ESE: 60 Marks Internal Assessment: 40 Marks	Theory: 3		
Tutorial: 1 Hour/Week			Tutorial: 1		
			Total: 4		
Course Prerequisite:					
	Basics of linear algebra, numerical methods, differential equations, and vector calculus.				
Course Objective: To develop the concept of					
1.	formulating the motion of space objects and visualization of different paths/orbits of				

	moving mass.		
2.	restricted problem, equilibrium points and their stability properties and application to real world problems.		
3.	perturbing factors in space, understanding and applications of different restricted few body problems along with computational techniques of orbits near equilibrium points.		
4.	canonical transformation, regularization of equations and normalization of Hamiltonian along with introduction to perturbation and KAM theory.		
Course O	utcomes: Students will be able to		
1.	formulate the motion of space objects and find out the paths/orbits of moving mass.		
2.	determine and examine the nature of equilibrium points in restricted three body problems and relate its application to real world problems.		
3.	know the impact of perturbing factors, understand the formulation of different restricted few body problems and know the methods of finding the orbits near equilibrium points.		
4.	know the use of canonical transformation, needs to regularize the equations of motion and application of normalized Hamiltonian in perturbation and KAM theory.		
Course Co	ontent:		
Unit-I	Introduction, Kepler's Laws of Planetary Motion, Newton's law of gravitation, Central force motion, Differential equation of orbit, Inverse square force, Geometry of orbits, Two body problem, Relative motion, Positions in different orbits. Uniform rotating	15 Hours	

	frame, Introduction to N-body problem, Mathematical formulation of N-body problem, Integrals of motion, Equation of relative motion.	
Unit-II	Three body problem, Existence of 10 integrals. Lagrange's straight line solution and Triangular solution and its applications, Restricted few body problem- formulation and its solution, Elliptic restricted three body problem. Lagrangian points and their Stability.	15 Hours
Unit-III	Formulation of Robe's RTBP, Hill's problem, Sitnikov problem and their applications, Different kinds of orbits and its families, Determination of orbits and their stability analysis, Differential correction method, Continuation technique, Poincare Lindstedt method, Impact of different perturbing factors on the motion of restricted mass.	15 Hours
Unit-IV	Regularization process and its uses, K-S regularization, Hamiltonian systems, Canonical transformations, Normal forms, normalization of Hamiltonian, Action-angle variables, Elementary ideas of perturbation theory and the KAM theory.	
Internal As	sessment:	1
CIA*-1	Written Exam.	
CIA-II	Written Exams/ Quizzes /Assignment /Presentations/ Viva-Voce	
ESE**	Written Exam. for Unit-I, II, III, & IV	
	us Internal Assessment Semester Examination	
Text Books	:	
	uskey S. W., 1963, Introduction to Celestial Mechanics, Adonpany.	lison-Wesley Publishing

- 2. Murray C. D. and Dermott S.F., 2000, Solar System Dynamics, Cambridge University Press.
- 3. Strogatz S.H., 1994, Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Chemistry and Engineering, Addison-Wesley.
- 4. Celletti A., 2010, Stability and Chaos in Celestial Mechanics, Praxis Publishing Ltd., Springer.
- 5. Rao K.S., 2009, Classical Mechanics, PHI Learning, Pvt. Ltd.

- 1. Moulton F.R., 1914, An Introduction to Celestial Mechanics, the MacMillan Company.
- 2. Szebehely V., 1967, Theory of orbits. The restricted problem of three bodies, New York Acad. Press.

### **E-resources:**

https://mitpress.mit.edu/9780262080484/celestial-mechanics/

<u>Course-Code:</u> MAT711 <u>Course Title:</u> ANALYTIC FUNCTIONS THEORY & SPECIAL FUNCTIONS				
Teaching SchemeExamination SchemeCredits Allotted				
Theory: 3 hours/ week	ESE: 60 Marks Internal Assessment: 40 Marks	Theory: 3		
Tutorial: 1 Hour/Week		Tutorial: 1		
		Total: 4		
Course Prerequisite:				

	Basics of Real and Complex Analysis.			
Course Obj	ective: To develop the concept of			
1.	The course presents an introduction to Geometric Function Theory including differential subordination, majorization and harmonic mappings. The aim of this course is to prepare the student for independent research in these topics.			
Course Out	comes:			
1.	The students will be equipped with the knowledge of basic theory and applications of Geometric Function Theory.			
Course Con	itent:			
Unit-I	Riemann Mapping theorem, Harmonic and subharmonic functions, Growth, covering and distortion theorems, Biberbach Conjecture, Littlewood theorem, Harglots representation theorem, Caratheodory Lemma, Starlike functions, Convex functions, Analytic and geometric descripritions, Alexander's theorem, Generalization of Koebe-1/4 theorem.	15 Hours		
Unit-II	Coefficient estimates, Close-to-Convex functions, Analytic and geometric descripritions, Radius of starlikeness and convexity, Subordination, Coefficient inequalities, Rogosinski Conjecture, Dieudonne's Lemma, Majorization, Sections of univalent functions, Convolution of convex functions, Marx-Strohhacker theorem,	15 Hours		
Unit-III	First and second order differential subordination, Briot- Bouquet differential subordination. Harmonic functions, Lewy's theorem, Heinz's Lemma, Rado's theorem, Shear Construction, Covering theorem and coefficient	15 Hours		

	bounds.		
Unit-IV	Schwarz Lemma for harmonic mappings, Harmonic Koebe function, Coefficient conjecture, Covering theorem, Harmonic starlike, Harmonic convex functions, Harmonic close-to-convex functions, Elementary properties.	15 Hours	
Internal A	ssessment:		
CIA*-1	Written Exam.		
CIA-II	Written Exams/ Quizzes /Assignment /Presentations/ Viva-Voce		
ESE**	Written Exam. for Unit-I, II, III, & IV		
	ous Internal Assessment Semester Examination		
Text Book	5:		
9. Gra Ma	ren P.L, 1983, Univalent Functions, Springer-Verlag. aham I. and Kohr G., 2003, Geometric Functions Theory in rcel Dekker. Iler S.S. and Mocanu P.T., 2000, Differential Subordination: T	C C	

- 10. Miller S.S. and Mocanu P.T., 2000, *Differential Subordination: Theory and Applications*, Marcel Dekker.
- 11. Duren P., 2004, Harmonic Mappings in the Plane, Cambridge University Press.

## **E-resources:**

## Course-Code: MAT712

Course Title: Advances in Fuzzy Set Theory, Fuzzy Logic and its Applications				
Teaching Scheme		Examination Scheme	Credits Allotted	
Theory: 3 hou	nrs/ week	ESE: 60 Marks Internal Assessment: 40 Marks	Theory: 3	
Tutorial: 1 Ho	our/Week		Tutorial: 1	
			Total: 4	
Course Prere	equisite: Stude	nts should have the knowledge of		
1.	Basics of clas	ssical sets, fuzzy sets and two-valued	logic.	
Course Obje	ctive:			
1.	To study generalized versions of fuzzy sets, which can capture more uncertainties and imprecise information in the world around us.			
2.	To explore applications of generalized version of fuzzy sets and fuzzy logic.			
Course Outn	Course Outmes: On completion of course, research students should be able to			
1.	think how generalized fuzzy sets can capture more uncertainty.			
2.	how concept of generalized fyzzy sets and fuzzy logic can be importent from teoretical and application point of view;			
3.	how generalized concept of fuzzy sets can be employed into different types of research problems involving imprecise and vague information.			
Course Content:				

Unit-I	Review of concept of Type-1 fuzzy sets, Intuitionistic fuzzy sets, Hesitant fuzzy sets, L-valued fuzzy sets, Interval-valued fuzzy sets, Type-2 fuzzy sets and their properties, Fuzzy functions, Fuzzy relational equations, Fuzzy relational systems.	15 Hours		
Unit-II	Review of Fuzzy logic and Fuzzy inference systems, Automated methods for fuzzy systems, Fuzzy controller: an overview and real life applications. Type-2 fuzzy logic systems with applications.	15 Hours		
Unit-III	Natural language, Linguistic variables, Computing with words, Computing with words and fuzzy logic, Computing with words and type-2 fuzzy sets. Fuzzy classification, Fuzzy pattern recognition.	15 Hours		
Unit-IV	Fuzzy decision making: Individual decision making, Multicriteria decision making, Ranking methods for fuzzy scenarios, Fuzzy linear programming problems. Fuzzy game theory.	15 Hours		
Internal Asso	essment:			
CIA*-1	Unit -I			
CIA-II	Written Exams/ Quizzes /Assignment /Presentations/ Viva-Voce			
ESE**	Unit-I,II,III,IV			
*: Continuous Internal Assessment **: End of Semester Examination				
Text Books:				
<ol> <li>Klir, G. J., Yuan B., 1997, Fuzzy Sets and Fuzzy Logic: Theory and Applications, Prentice Hall.</li> <li>Belohlavek R., 2002, Fuzzy Relational Systems: Foundations and Principles, Springer Science+Business Media, New York.</li> <li>Chaira T., 2019, Fuzzy Set and its Extension: The Intuitionistic Fuzzy Set, Wiley.</li> </ol>				

4. Sadeghian A., Mendel J.M. and Tahayori H., 2013, *Advances in Type-2 Fuzzy Sets and Systems: Theory and Applications*, New York, USA, Springer.

- 1. Zadeh L. A., 2012, *Computing with Words—Principal Concepts and Ideas*, Stud. Fuzziness Soft Comput., Vol. 277, Springer, Berlin.
- 2. Oscar Castillo, Patricia Melin, *Type-2 Fuzzy Logic: Theory and Applications*, Studies in Fuzziness and Soft Computing (STUDFUZZ, volume 223), Springer.

### **E-resources:**

1. https://archive.nptel.ac.in/courses/108/104/108104157/

<u>Course-Code: MAT-713</u> <u>Course Title:</u> MAGNETOHYDRODYNAMICS			
Teaching Scheme	Examination Scheme	Credits Allotted	
Theory: 3 Hours/week	ESE: 60 Marks Internal Assessment: 40 Marks	Theory: 3	
Tutorial: 1 Hour/week		Tutorial: 1	
		Total: 4	

Course ]	Prerequisite: The student should have knowledge of	
1.	basic concepts of fluid dynamics.	
2.	basic concepts of calculus.	
Course	Objective: This course aims to learn	
1.	the fundamentals of MHD flow and electrically conducting fluids.	
2.	the governing equations of Magnetohydrodynamics.	
3.	the governing equations in different coordinate systems and boundary conditions.	
4.	the boundary layers theory in Magnetohydrodynamics.	
Course	Dutcomes: After studying this course the student will be able to	
1.	basic properties of electrically conducting fluids	
2.	understand the fundamentals of MHD	
3.	the solution of fundamental equations of MHD	
4.	applications of MHD to various science and engineering fields	
Course Content:		

Unit-I	Basic concepts of Magnetohydrodynamics and its applications, Maxwell's equations, Frame of reference, Ampere's Law, Faraday's Law in differential form, Lorentz force, Electromagnetic body force, Ohm's law for a moving conductor, Hall current, Conduction current, Magnetic Reynolds number.	15 Hours
Unit-II	Kinematic aspect of MHD, Navier-Stokes equations incorporating of Lorentz forces in different co-ordinate systems,	15 Hours
Unit- III	Governing Equations of MHD: MHD heat transfer, magnetic field equations, induced magnetic field equations with high and low magnetic Reynolds number.	15 Hours
Unit- IV	Hartmann flow, Hartmann boundary layer, Hartmann flow between two planes, Couette flow, Couette boundary layer, Couette flow between two planes, MHD Stoke's flow, MHD Rayleigh's flow, Aligned flow in two dimensional MHD flow, MHD flow in rotating medium.	15 Hours
Internal A	Assessment:	
CIA-I*	Unit-I	
CIA-II	Written Exams/ Quizzes/ Assignment/ Presentations/ Viva-Voce	
ESE**	Unit-I, II, III, IV	
	uous Internal Assessment f Semester Examination	
Text Bool	xs:	
1. Cov	vling T. G., 1957, <i>Magnetohydrodynamics</i> , Interscience Publishers.	

2. Shercliff J. A., 1965, A Text Book of Magnetohydrodynamics, Pergamon Press.

3. Pai S. I., 1962, Magnetohydrodynamics and Plasma Dynamics, Springer.

4. Cramer K. R. and Pai S. I., 1973, *Magnetofluid Dynamics for Engineers and Applied Physicists*, McGraw Hill.

**Reference Books:** 

**E-resources:** 

<u>Course-Code: MAT 714</u> <u>Course Title: MATHEMATICAL BIOLOGY</u>			
Teaching Scheme	Examination Scheme	Credits Allotted	
Theory: 3 hours/ week	ESE: 60 Marks Internal Assessment: 40 Marks	Theory: 3	

Tutorial: 1 Hour/Week					Tutorial: 1
					Total: 4
Course Pr	rerequisite: The stu	ident should have kno	owledge of		
1.	Analysis, Linear A	Algebra and Different	ial Equations		
Course O	bjective:				
1	To explain real challenges of mathematical modelling in biology				
2	to introduce basic assumptions and hypotheses for the mathematical formulations of a biological system				
3	to introduce different types of models in biology and how to predict its different dynamical behaviour				
4	To discuss the dynamical analysis of different models using linearization, stability analysis and bifurcation theory				
5	To introduce the modeling of chemical kinetics				
Course O	Course Outcomes: The students will be able to understand				
1.	11.	ults from dynamical s analyze a given biolo	ystems, differential ec gical system	quations, b	ifurcation and
2.	and develop essential theory and basics of modeling the various types of interactions among species				
3.	modeling and and interacting popula	•	mpartmental epidemic	c models,	single species,

4.	how mathematics can help to predict certain complex behaviour (bifu solutions, dynamics in future time etc.) of the associated system	rcations, stability of
Course C	ontent:	
Unit-I	Introduction: Goals and Challenges of mathematical modeling in biology and ecology. Idealization and general principle of model building, deterministic and stochastic models, different types of mathematical models and differential and difference equations as relevant mathematical techniques, complex network dynamics, biological and ecological examples	15 Hours
Unit-II	spatial and non-spatial models, time-discrete and time-continuous models, The Single Species population models (discrete and continuous): Exponential, Logistic, and Gompertz growth, Allee effect, Harvesting models and bifurcations, Delay models, Models with interacting populations: Different types of interactions and examples, Lotka Volterra Competition, Predator-prey model, Chemostat models	15 Hours
Unit-III	Structured (spatial, age and sex) population models, metapopulation and metacommunity dynamics, Modeling applications: biological invasions, biological pattern formations, species succession, biodiversity maintenance mechanisms, The population biology of infectious diseases: Classification of infectious diseases, SIR, SIRS and SIS epidemic models, Basic reproduction number	15 Hours
Unit-IV	Models for molecular events: Michaelis-Menten enzyme example, Timescale decomposition, Quasi steady state analysis, sigmoidal functions, multisite systems, Chemical kinetics: Mass action law, Hopf-bifurcations, Subcritical Hopf, Poincare-Bendixson-I, Poincare- Bendixson-II, Index Theory.	15 Hours
Internal A	Assessment:	
CIA*-1	Unit -I	
CIA-II	Written Exams/ Quizzes /Assignment /Presentations/ Viva-Voce	

LOL				
*: Continuous Internal Assessment **: End of Semester Examination				
Text Books:				
1. Brauer F. and Chavez C. C., 2000, Mathematical Models in Population Biology and Epidemiology, Springer.				
2. Ko	t M., 2001, Elements of Mathematical Ecology, Cambridge University Press.			
3. Ke	3. Keshet L. E., 2005, Mathematical Models in Biology, SIAM.			
4. Keeling M. J. and Rohani P., 2008, <i>Modeling Infectious Diseases in Humans and Animals</i> , Princeton University Press.				
5. Ma	5. Martcheva M., 2010, An Introduction to Mathematical Epidemiology, Springer.			
6. Bri	itton N. F., 2004, Essential Mathematical Biology, Springer.			
Reference	e Books:			
8. Sr Sc	urray J. D., 2007, Mathematical Biology: An Introduction, Springer. nith H., 2010, An Introduction to Delay Differential Equations with Applications to Life iences, Springer. kubo A. and Levin, S. A., 2002, Diffusion and Ecological Problems, Springer.			

## **E-resources:**

Unit-I, II, III, IV

ESE\*\*

https://open.uci.edu/courses/math\_113b\_intro\_to\_mathematical\_modeling\_in\_biology.html

https://www.youtube.com/playlist?list=PL5zWDs2j0YF3kPPvs4L5FGlLc7x13Uwjn

Course Code: MAT715			
<b>Course Title:</b> NUMERICAL METHODS FOR HYPERBOLIC PDES			
Teaching Scheme	Examination Scheme	Credits Allotted	
Theory: 3 hours/ week	ESE: 60 Marks Internal Assessment: 40 Marks	Theory: 3	
Tutorial: 1 Hour/Week		Tutorial: 1	

				Total: 4	
Course Prerequisite: Numerical Methods. Partial Differential Equations					
Course O	Course Objective:				
1	This course introduces the system of hyperbolic conservation laws;				
2	It discuss the existence and uniqueness of the solutions for hyperbolic conservation laws.				
3	Further it describes the first-order and higher order numerical methods to solve these systems.				
Course Outcomes: The students will be able to understand					
1.	To construct first order numerical schemes for the given system of hyperbolic PDEs				
2	To develop high o	rder numerical schemes for h	yperbolic PDEs.		
3	The students will be able to solve the hyperbolic PDEs numerically.				
Course Content:					
Unit-I	problem, Classica Rankine-Hugonio	on law: Hyperbolic conse l solution, Method of charact c condition, Shocks and Rare and uniqueness results.	eristic, Weak solution,	15 Hours	
Unit-II	schemes, Higher	emes, Riemann solvers, Con -order schemes: Lax-Wenc s, ENO schemes, Higher-ord	droff schemes, TVD	15 Hours	

ear systems: Exact solutions, first-order and higher-order finite ume schemes. Nonlinear systems: solutions of Riemann problems, t-order finite volume schemes for systems. her-order schemes for systems: TVD limiters, finite-volume emes on unstructured meshes. Hyperbolic systems with source ns. sment: t -I tten Exams/ Quizzes /Assignment /Presentations/ Viva-Voce	15 Hours
emes on unstructured meshes. Hyperbolic systems with source ns. sment: t -I	
t -I	
tten Exams/ Ouizzes /Assignment /Presentations/ Viva-Voce	
alon Dhams, Quillos (1188) ginnone (1188) endalons, (114 (886)	
t-I,II,III,IV	
Internal Assessment lester Examination	1
. C., 1998, Partial Differential Equations, AMS.	
F., 2009, <i>Riemann Solvers and Numerical Methods for Fluid Dy</i> action (3 <sup>rd</sup> Ed.), Springer.	namics: A Practical
ks:	
]	ester Examination . C., 1998, Partial Differential Equations, AMS. F., 2009, Riemann Solvers and Numerical Methods for Fluid Dy ction (3 <sup>rd</sup> Ed.), Springer.

2. Godlewski E. and Raviart P. A., 1991, Hyperbolic Systems of Conservation Laws,

Mathematiques & Applications.

**E-resources:** 

<u>Course-Code: MAT716</u> <u>Course Title: QUEUEING THEORY</u>				
Teaching	Scheme	Examination Scheme	Credits Allotted	
Theory: 3 hours/ week		ESE: 60 Marks Internal Assessment: 40 Marks	Theory: 3	
Tutorial: 1 Hour/Week			Tutorial: 1	
			Total: 4	
Course P	Course Prerequisite: The student should have a good knowledge of			
1.	probability theory			
Course O	Course Objective:			
1	To teach theory of queues.			
2	To teach them Markovian property, various models related to queueing problems.			

3	To teach Erlangs model. Priority queues, Retrial queues, General arriva patterns.	al and service		
Course O	utcomes: The students will be able to understand			
1.	Students will learn various aspects of queueing theory			
2.	They will be able to develop mathematical models for various practical problems arising in day today life.			
3.	Students will be able to calculate various performance measures of the model and cost analysis of the problem.			
Course C	Course Content:			
Unit-I	Introduction to probability and general rules, stochastic processes, Stationary Processes: Weakly stationary and strongly stationary processes, Non-Markov processes, Markov processes, Discrete-time Markov Chains: Transition probability matrix, Chapman-Kolmogorov equations; n-step transition and limiting probabilities, ergodicity, stationary distribution, random walk and gambler's ruin problem, applications of DTMCs.	15 Hours		
Unit-II	Continuous-time Markov Chains (CTMCs): Kolmogorov differential equations for CTMCs, infinitesimal generator, stationary and transient distribution, Poisson process. Queueing models, pure birth model, pure death model, birth-death model,	15 Hours		
Unit-III	M/M/1 model, M/M/c model, M/M/c/K model, M/M/c/∞ model. Renewal Processes: Renewal function and	15 Hours		

	its properties, renewal theorems, cost/rewards associated with renewals, Markov renewal and regenerative processes.		
Unit-IV	Unit-IV Embedded Markov chains, General arrival and service processes, M/G/1, G/M/1, M/G/c/∞, queueing models, Semi Markov process and Markov renewal processes in queueing. Erlang model and loss system. Retrial queues. Stochastic Petri net, applications to queueing theory and communication networks.		
Internal A	Assessment:		
CIA*-1	Unit -I		
CIA-II	Written Exams/ Quizzes /Assignment /Presentations/ Viva-Voce		
ESE**	Unit-I, II, III, IV		
	nous Internal Assessment f Semester Examination		
Text Bool	ks:		
<ol> <li>Bertsekas D. and Tsitsiklis J., 2008, <i>Introduction to Probability (2<sup>nd</sup> Ed.)</i>, Athena Scientific.</li> <li>Castaneda L. B., Arunachalam V. and Dharmaraja S., 2012, <i>Introduction to Probability and Stochastic Processes with Applications</i>, Wiley, Hoboken, NJ, USA.</li> <li>Gross D., Shortle J.F., Thompson J.M. and Harries G.M., 2018, <i>Fundamental of Queueing Theory</i>, John Wiley &amp; Sons, Inc, Hoboken, NJ.</li> <li>Trivedi K.S., 2016, <i>Probability and Statistics with Reliability, Queuing and Computer Science Applications</i>, John Wiley &amp; Sons, Inc., Hoboken, NJ, USA.</li> </ol>			

5. Nelson R., 2013, *Probability, Stochastic Processes, and Queueing Theory: The Mathematics of Computer Performance Modeling*, Springer Science & Business Media.

## **E-resources:**

1. https://archive.nptel.ac.in/courses/111/102/111102111/

Course Code: MAT717 <u>Course Title:</u> HOLOMORPHIC DYNAMICS			
Teaching Scheme     Examination Scheme     Credits Allotte		Credits Allotted	
Theory: 3 hours/ week		ESE: 60 Marks Internal Assessment: 40 Marks	Theory: 3
Tutorial: 1 Hour/Week			Tutorial: 1
			Total: 4
Course Prerequisite: Complex analysis			
Course	<b>Objective:</b> To intro	oduce	
1	Escaping points	Escaping points of polynomials	
2	Hausdorff dime	Hausdorff dimension	

3	Snail Lemma of Fatou set		
Course O	utcomes: The students will be able to understand		
1.	Mandelbrot sets		
2	Julia set and the Fatou set		
3	Nevanlinna inequality		
4	special classes of entire functions		
Course C	ontent:		
Unit-I	Escaping points of polynomials, Basic properties of filled Julia set, local behavior near fixed point, Quadratic polynomials and the Mandelbrot sets,	15 Hours	
Unit-II	Hausdorff dimension and capacity, Polynomial Like mappings, Hyperbolic domains, contraction principle, Normal families, Julia set and the Fatou set, classification theorem,	15 Hours	
Unit-III	Snail Lemma of Fatou set, Value distribution theory, The 2nd main theorem and direct consequence, Nevanlinna inequality, basic properties of the Julia set, The first fundamental theorem, Rescaling Lemma of Zalcman, singular values and Fatou components,	15 Hours	
Unit-IV	The second fundamental theorem, topological properties of Fatou set and the Julia set, special classes of entire functions, Escaping points of transcendental entire and meromorphic functions.	15 Hours	
Internal A	Assessment:		

CIA*-1	Unit -I			
CIA-II	Written Exams/ Quizzes /Assignment /Presentations/ Viva-Voce			
ESE**	Unit-I,II,III,IV			
	ous Internal Assessment Semester Examination			
Text Bool	xs:			
<ol> <li>Morosawa S., Nishimura Y., Tanuguchi M. and Ueda T., 2000, <i>Holomorphic Dynamics:</i> <i>Cambridge studies in Advanced Mathematics</i>, Cambridge University Press,.</li> <li>Hua X. H. and Yang C. C., 1998, <i>Dynamics of Transcendental Functions</i>, Gordon and Breach Science Pub.</li> </ol>				
Reference	e Books:			
5. Milnor J., 2006, <i>Dynamics in One Complex Variable (3<sup>rd</sup> Ed.)</i> : Annals of Mathematics Studies, Princeton University Press				
E-resources:				

# <u>Course-Code: MAT 718</u> Course Title: APPLIED DYNAMICAL SYSTEMS

Teaching Scheme		Examination Scheme	Credits Allotted
Theory: 3 hours/ week		ESE: 60 Marks Internal Assessment: 40 Marks	Theory: 3
Tutorial: 1 Hour/Week			Tutorial: 1
			Total: 4
Course Prere	equisite: The st	udent should have knowledge of	
1.	Differential E	quations, Analysis and Linear Algebra	a and Real Analysis, Geometry
Course Obje	ctive:		
1.	To present the poincare map and its applications for stability behavior.		
2.	To illustrate different bifurcations and their underlying mathematical theory		
3.	To introduce Hartman-Grobman theorem and stable manifold theorem		
4.	To discuss a variety of methodologies for the analysis of transient and chaotic dynamics.		
5.	To introduce different aspects associated with discrete dynamical systems		
Course Outco	omes: The stud	ents will be able to	
1.	construct and apply poincare maps and doing bifurcation analysis analytically.		
2.	work with m	aps and analyze their chaotic character	ristics if they exist.
3.	learn different dynamical systems tools e.g., Hartman-Grobman, stable manifold theorems, center and normal form theory		

4.	To analyze discrete dynamical systems and their stability		
5.	how to apply these concepts in different fields like nonlinear dynamics, population dynamics, chemical kinetics, epidemiology, environmental sciences etc.		
Course Cont	ent:		
Unit-I	Dynamical systems: continuous and discrete. Poincare surface of section, limit sets, attractors, basin of attraction	15 Hours	
Unit-II	Sensitive dependence on parameters and initial conditions, One-dimensional maps, fixed points, periodic orbits and their stability, logistic map, bifurcations, Universal scaling of period doubling bifurcations in quadratic maps,	15 Hours	
Unit-III	other types of bifurcations in one-dimensional maps, Lyapunov exponents, Quasi-periodicity, circle map, Hopf bifurcations, Hamiltonian systems, Symplectic structure, Canonical transformations, integrable systems, perturbation of integrable systems	15 Hours	
Unit-IV	Kolmogorov-Arnold-Moser theorem, resonant tori, Chaotic transitions, intermittency, crises. Time series of discrete dynamical systems: system evolution and attractor, correlation integral, estimators of correlation integral, Takens theorem	15 Hours	
Internal Asse	essment:		
CIA*-1	Unit -I		
CIA-II	Written Exams/ Quizzes /Assignment /Presentations/ Viva-Voce		
ESE**	Unit-I, II, III, IV		
*: Continuous Internal Assessment **: End of Semester Examination			

### **Text Books:**

1. Perko L., 2006, Differential Equations and Dynamical Systems, Springer-Verlag.

2. Stuart A. M. and Humphries A. R., 1998, *Dynamical Systems and Numerical Analysis*, Cambridge University Press.

3. Wiggins R.L., 1997, Introduction to Applied Nonlinear Dynamical Systems, Springer Verlag.

4. Lynch S., 2004, Dynamical Systems with Applications using MATLAB, Birkhause Press.

5. Alligood K., Sauer T.D. and Yorke J.A., 1996, *Chaos: An Introduction to Dynamical Systems,* Springer Verlag.

6. Ott, E., 2002, Chaos in Dynamical Systems, Cambridge University Press.

#### **Reference Books:**

- 1. Strogatz, S. H., 2000, Nonlinear Dynamics and Chaos with Applications to Physics, Biology, Chemistry and Engineering, Westview Press.
- 2. Devaney, 2003, An Introduction to Chaotic Dynamical Systems, Addison-Wesley.

**E-resources:** 

https://www.youtube.com/playlist?list=PLbN57C5Zdl6j\_qJA-pARJnKsmROzPnO9V

https://www.youtube.com/watch?v=BRaliLNuvNg&list=PL6hB9Fh0Z1ELbHIAL22dCk173qykDgeoz

<u>Course-Code: MAT-719</u> <u>Course Title:</u> THERMAL INSTABILITIES AND METHODS					
Teaching S	Scheme	Examination Scheme	Credits Allotted		
Theory: 3 Hours/week		ESE: 60 Marks Internal Assessment: 40 Marks	Theory: 3		
Tutorial: 1	1 Hour/week		Tutorial: 1		
			Total: 4		
Course Pr	erequisite: The stu	ident should have knowledge of			
1.	basic concepts of fluid dynamics.				
2.	basic concepts of numerical methods.				
Course Objective: This course aims to learn					

1.	the fundamental understanding of the instabilities observed in fluid and therma	al systems.
2.	the thermal instability transition in fluid flow.	
3.	some methods to solve the instability problems.	
Course Ou	itcomes: After studying this course the student will be able to	
1.	basic properties of the instability process in fluid and thermal systems.	
2.	the truncated representation of the Fourier series.	
3.	solution methods to solve the instability problems.	
Course Co	ontent:	
Unit-I	Rayleigh-Benard convection, oberbeck convection, Marangoni convection, and mechanism of instability, Benard and Taylor instabilities, and various types of convection instabilities.	15 Hours
Unit-II	Magneto-convection, magneto-Marangoni convection, electro-convection, double diffusive convection, cross diffusion convection, bi-convection, different type of boundary conditions.	15 Hours
Unit- III	Techniques to solve linear and nonlinear instability problems; Galerkin technique, perturbation techniques involving regular and singular perturbations	15 Hours
Unit- IV	Truncated representation of Fourier series (finite amplitude technique), numerical techniques, moment method, energy method, power integral technique, spectral method.	15 Hours

	Assessment:	
CIA-I*	Unit-I	
CIA-II	Written Exams/ Quizzes/ Assignment/ Presentations/ Viva-Voce	
ESE**	Unit-I, II, III, IV	
	uous Internal Assessment f Semester Examination	
Text Bool	S:	
1. Draz Pres	in P. G. and Reid W. H., 2004, <i>Hydrodynamic Stability</i> , (Ed. 2 <sup>nd</sup> ), Cambridgs.	ge University
2. Cha	ndrasekhar S., 1961, Hydrodynamic and Hydromagnetic Stability, Oxford Univ	ersity Press.
3. Niel	d D. A. and Bejan A., 2006, Convection in Porous Medium, Springer.	
4. Shiva McGraw-1	kumara I. S. and Venkatachalappa M., 2004, Advances in Fluid Mechanics, Vo Hill.	<i>l 4</i> , Tata
Reference	Books:	

Course Code: MAT720

Course Title: COMPLEX DYNAMICS

Teaching	g Scheme	Examination Scheme	Credits Allotted	
Theory: 3	3 hours/ week	ESE: 60 Marks Internal Assessment: 40 Marks	Theory: 3	
Tutorial:	1 Hour/Week		Tutorial: 1	
			Total: 4	
Course P	Prerequisite: Comp	plex analysis	1	
Course C	<b>Dbjective:</b> To introd	duce		
1	Iteration of a Mo	bius transformation,		
2	Lipschitz condition.			
3	Normal families			
Course (	<b>Dutcomes:</b> The stud	lents will be able to understand		
1.	chordal metric			
2	Riemann Hurwit	Riemann Hurwitz relation		
3	minimal property of Julia sets			
4	The Euler characteristic			
Course (	Content:			
Unit-I	indifferent fixed	Iteration of a Mobius transformation, attracting, repelling and indifferent fixed points. Iterations of $R(z) = z^2 2$ , $z + c$ , $z + .$ The extended complex plane, chordal metric, spherical metric, rational 15 Hours		

maps,	
Lipschitz condition, conjugacy classes of rational maps, valency of a function, fixed points, Critical points, Riemann Hurwitz relation, Equicontinuous functions, normality sets, Fatou sets and Julia sets,.	15 Hours
completely invariant sets, Normal families and equicontinuity, Properties of Julia sets, exceptional points backward orbit, minimal property of Julia sets.	15 Hours
Julia sets of commuting rational functions, structure of Fatou set, Topology of the Sphere, Completely invariant components of the Fatouset , The Euler characteristic, Riemann Hurwitz formula for covering maps, maps between components of the Fatou sets, the number of components of Fatou sets, components of Julia sets	15 Hours
Assessment:	
Unit -I	
Written Exams/ Quizzes /Assignment /Presentations/ Viva-Voce	
Unit-I,II,III,IV	
uous Internal Assessment f Semester Examination	
ks:	
. F. Beardon, 1991, Iteration of rational functions, Springer Verlag, New	York. rlag.
	Lipschitz condition, conjugacy classes of rational maps, valency of a function, fixed points, Critical points, Riemann Hurwitz relation, Equicontinuous functions, normality sets, Fatou sets and Julia sets,. completely invariant sets, Normal families and equicontinuity, Properties of Julia sets, exceptional points backward orbit, minimal property of Julia sets.  Julia sets of commuting rational functions, structure of Fatou set, Topology of the Sphere, Completely invariant components of the Fatouset , The Euler characteristic, Riemann Hurwitz formula for covering maps, maps between components of Julia sets  Assessment: Unit -I Unit -I Unit -I Unit -I,II,III,IV  Loous Internal Assessment

2. 4. X. H. Hua, C. C. Yang, 1998, *Dynamics of transcendental functions*, Gordan and Breach Science Pub.

**E-resources:** 

<u>Course-Code: MAT 721</u> Course Title: FRACTURE MECHANICS			
Teaching	Examination Scheme	Credits Allotted	

Scheme				
Theory: week	Theory: 3 hours/ ESE: 60 Marks Theory: 3 Internal Assessment: 40 Marks			
	Tutorial: 1     Tutorial: 1       Hour/Week     Tutorial: 1			
			Total: 4	
Course ]	Prerequisi	te: The student should have knowledge of		
1.	Basics of	f Mechanics		
Course	Objectives	:		
1.	To intro	duce the mechanics of anisotropic material		
2.	To provide insight into different failure mechanisms.			
3.	To provide insight of smart materials and their fracture mechanism			
Course	Outcomes	The students will be able to		
1.	develop a	a deep understanding of the failure of materials		
2.	capable of applying these concepts to obtain the solutions to the fracture mechanics problems			
Course	Content:			
Unit-I	Introduction to the realm of fracture and background history of development of fracture mechanics; Discrepancy between theoretical and real strength of materials, conventional failure criteria based on stress concentration and characteristic brittle failures, Griffith's work.		15 Hours	
Unit-II			15 Hours	

Unit-III	Elastic-Plastic Fracture Mechanics Based Design Criteria: Design criteria for non-brittle materials; plastic zone corrections, crack opening displacement (COD), J-contour integral and crack growth resistance (R-curve) concepts.	15 Hours
Unit-IV	Introduction to smart materials, Piezoelectrics and Magneto Electro Elastic materials, review of crack problems in these materials.	15 Hours
Internal	Assessment:	
CIA*-1	Written exam	
CIA-II	Written Exams/ Quizzes /Assignment /Presentations/ Viva- Voce	
ESE**	Unit-I, II, III, IV	
	nuous Internal Assessment of Semester Examination	
Text Bo	oks:	
	Gdoutos, E.E., 2005, Fracture Mechanics: An Introduction, 2nd E Kumar, P., 1999, Elements of Fracture Mechanics, Wheeler Publi	-
Referen	ce Books:	
		unlighting and Ed. CDC
	nderson, T. L., 2005, <i>Fracture Mechanics: Fundamentals and A</i> Press.	ppucations, sid Ed., CRC

Course Code: MAT722			
	<u>Course Title:</u>	Fuzzy Automata and Languages: Theory and Appl	lications
Teaching	Scheme	Examination Scheme	Credits Allotted
Theory: 3	hours/ week	ESE: 60 Marks Internal Assessment: 40 Marks	Theory: 3
Tutorial: 1	Hour/Week		Tutorial: 1
			Total: 4
Course P	rerequisite: Basic	s of automata and languages	
Course O	bjective:		
1	To develop thinking about the usefulness of uncertainty in the world around us.		
2	To explain how a machine can capture uncertainty and vagueness involved in a system environment.		
3	To introduce fuzziness in automata theory and languages.		
Course Outcomes: The students will be able to understand			
1.	how natural languages lack the precision of formal languages.		
2	how introduction of fuzziness into the structure of formal languages will help to reduce the gap between formal language and natural languages.		

3	how fuzziness is helpful in the development of a fuzzy machine to make	e it more applicable.	
4	the real life examples of application of fuzzy machines and fuzzy langua	ages.	
Course C	ontent:		
Unit-I	Review of finite-state machine, Finite-state automata, Language and Grammar, Non-deterministic automata, Relationship between languages and Automata, Pushdown automata, Lattices, complete lattices, residuated lattices and their properties, Generalized residuated lattices and their properties. Fuzzy Machine, Language and Grammars, Fuzzy Languages and Grammars	15 Hours	
Unit-II	Algebraic Fuzzy automata theory: Fuzzy finite-state machines, Semigroup of fuzzy finite-state machines, Homomorphisms, Admissible relations, Fuzzy transformation semigroups, Product of fuzzy finite-state machines, Submachines of a fuzzy finite-state machine, Retrievability, separability and connectivity, Decomposition of finite-state machines.	15 Hours	
Unit-III	Subsystems of fuzzy finite-state machines, Strongsubsystems, Covering of Product of fuzzy finite-state machines, Covering properties of Products, Fuzzy regular languages, Fuzzy recognizers, Minimal fuzzy recognizers, Myhill-Nerode theorem for fuzzy languages.	15 Hours	
Unit-IV	Equivalence, Reduction and minimization of finite fuzzy automata, Minimal fuzzy finite-state automata, Behaviour, Reduction and minimization of finite L-automata, fuzzy languages based on different structures. Minimal realizations of fuzzy languages:different approaches, Applications of fuzzy automata and languages.	15 Hours	
Internal Assessment:			

CIA*-1	Unit -I			
CIA-II	Written Exams/ Quizzes /Assignment /Presentations/ Viva-Voce			
ESE**	Unit-I,II,III,IV			
*: Continuous Internal Assessment **: End of Semester Examination				
Text Books:				

- 6. John N. Mordeson, D.S.Malik, Fuzzy Automata and Languages: Theory and Applications, CRC Press, Taylor and Francis Group, New York.
- 7. Adamek J., Herrlich H. and Strecker G. E., 2006, *Abstract and Concrete Categories: The Joy* of *Cats*, Dover Publications.

8. Arbib M. A. and Manes E. G., 1975, Arrows, Structures & Functors: The Categorical Imperative, Academic Press..